

# Multi-particle eccentricities in A+A and p+A collisions

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based on:

AB, P. Bozek, L. McLerran, arXiv:1311.7325

L. Yan, J.-Y. Ollitrault, arXiv:1312.6555

AB, V. Skokov, arXiv:1312.7349

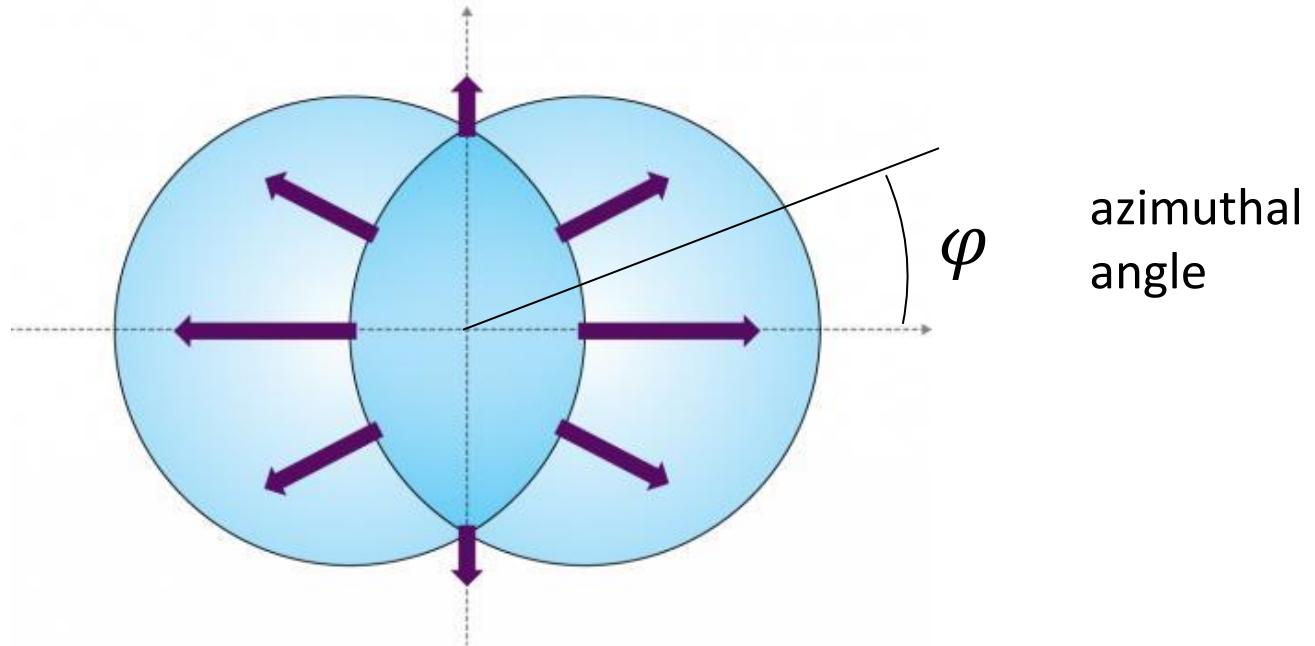
## Outline

- introduction
- multi-particle cumulants
- A+A collisions
- p+A collisions
- conclusions

# Elliptic flow, ellipticity

J.-Y. Ollitrault, PRD 46 (1992) 229

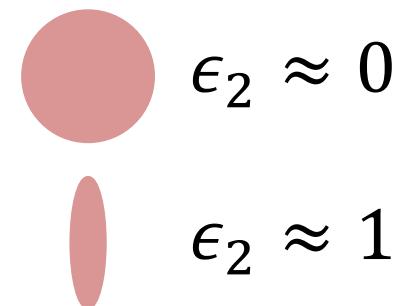
transverse plain  
of a heavy-ion  
collision



$$\frac{dN}{d\varphi} \sim 1 + 2v_2 \cos(2\varphi) + \dots$$

Hydrodynamics:

$$v_2^2 \sim \epsilon_2^2 = \frac{\left[ \sum_i r_i^2 \cos(2\varphi_i) \right]^2 + \left[ \sum_i r_i^2 \sin(2\varphi_i) \right]^2}{\left[ \sum_i r_i^2 \right]^2}$$



## Correlations:

N.Borghini, P.M.Dinh, J.-Y.Ollitrault, PRC 63 (2001) 054906

$$\begin{aligned} (\nu_2\{2\})^2 &= \langle e^{i2(\varphi_1 - \varphi_2)} \rangle & c_m \text{ is non-flow} \\ &= \langle \nu_2^2 \rangle + c_2 \end{aligned}$$

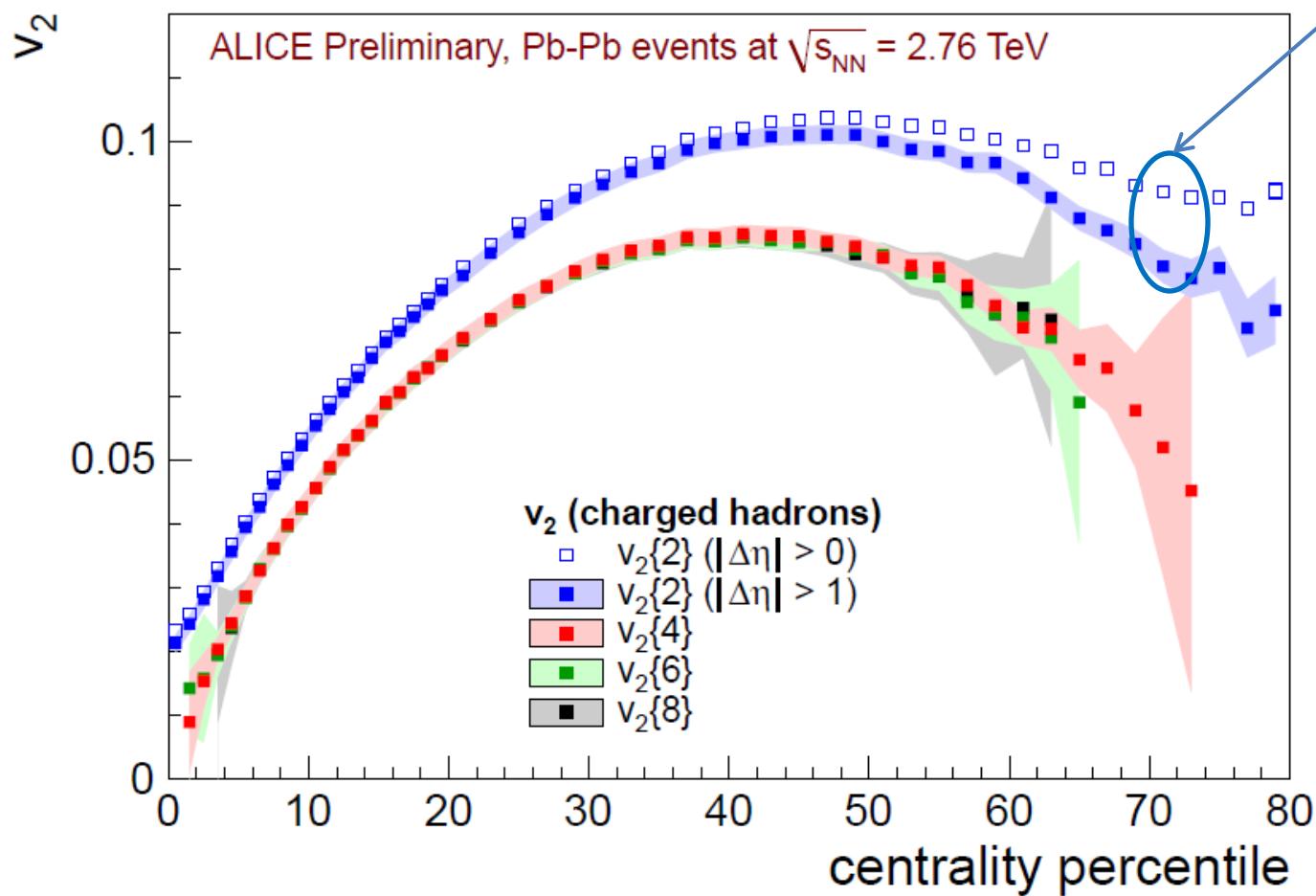
$$\begin{aligned} (\nu_2\{4\})^4 &= -\langle e^{i2(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \rangle + 2\langle e^{i2(\varphi_1 - \varphi_2)} \rangle^2 \\ &= -\langle \nu_2^4 \rangle + 2\langle \nu_2^2 \rangle^2 + c_4 & \text{etc.} \end{aligned}$$

If in each event  $\nu_2 = \bar{\nu}_2$ :

$$\begin{aligned} \nu_2\{2\} &= \bar{\nu}_2 + c_2 \\ \nu_2\{m\} &= \bar{\nu}_2 + c_m \end{aligned}$$

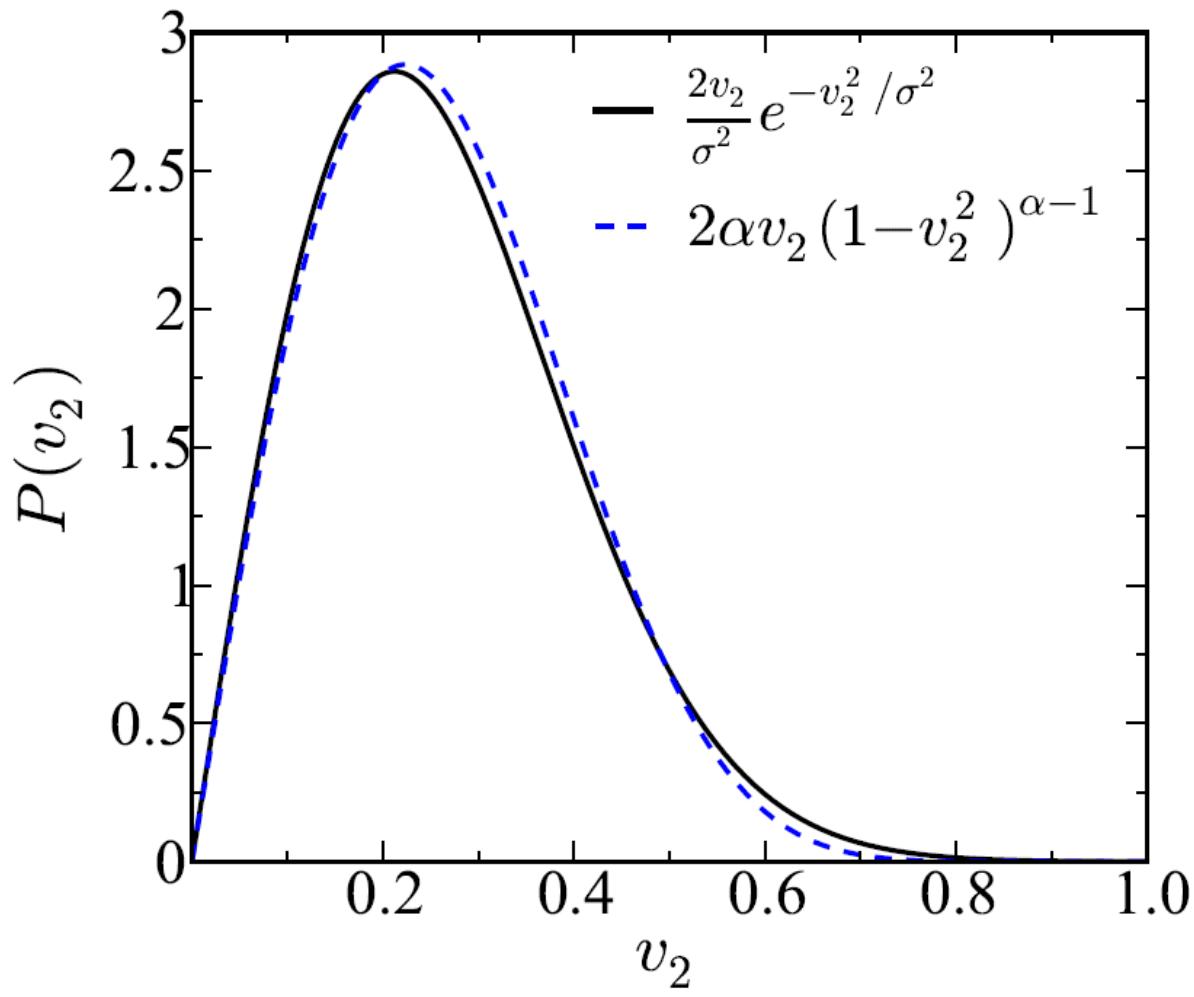
# Pb-Pb, is it obvious?

this difference is  
due to  $c_2$



difference between  $v_2\{2\}$  and  $v_2\{4\} \approx v_2\{6\} \approx v_2\{8\}$  is due to flow fluctuations but ...

# multi-particle moments are not trivial



In this plot:  $\sigma = 0.3$ ,  $\alpha = 10.5$

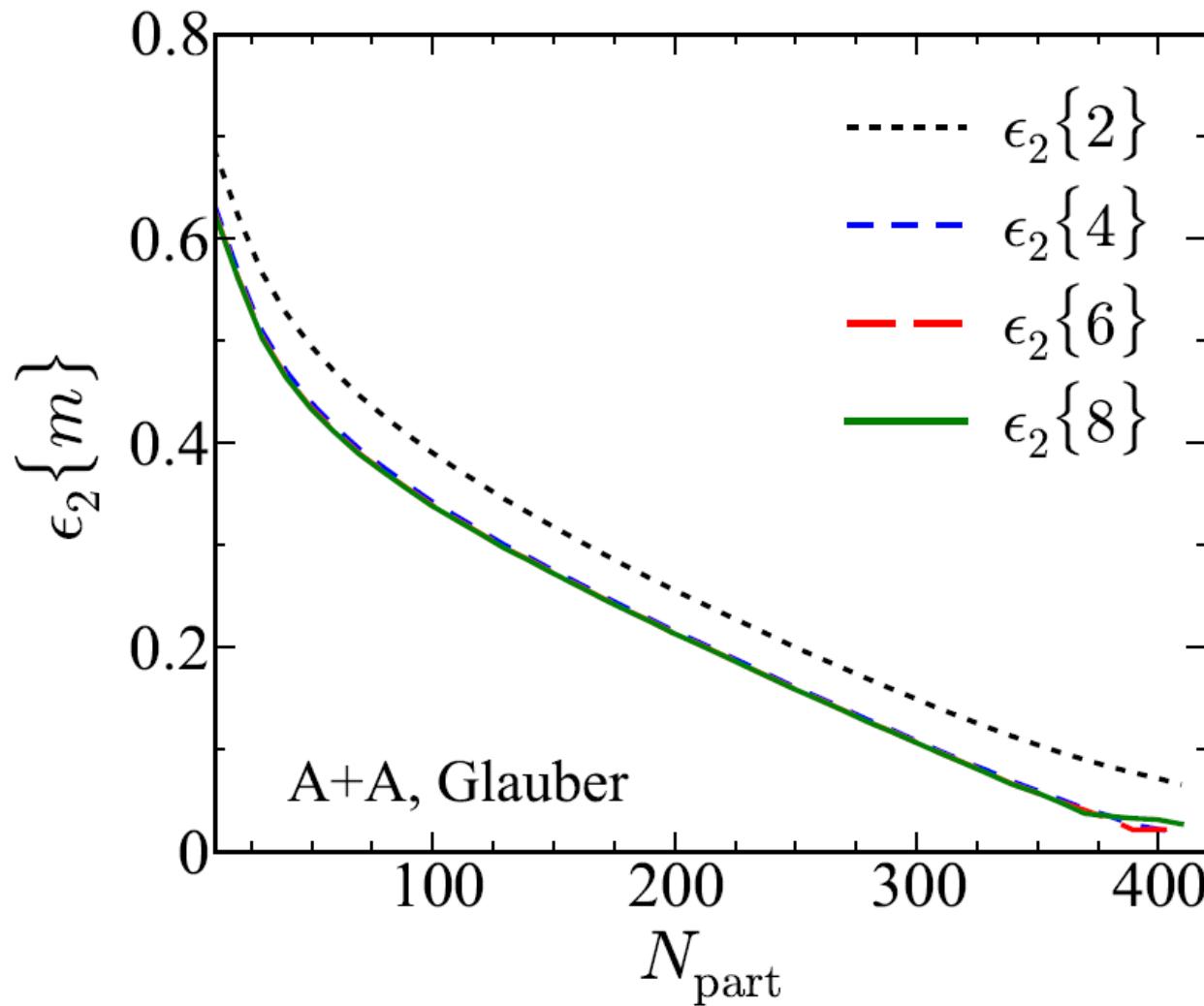
Gauss:

$$\begin{aligned}v_2\{2\} &= \sigma \\v_2\{4\} &= 0 \\v_2\{6\} &= 0 \\v_2\{8\} &= 0\end{aligned}$$

Power:

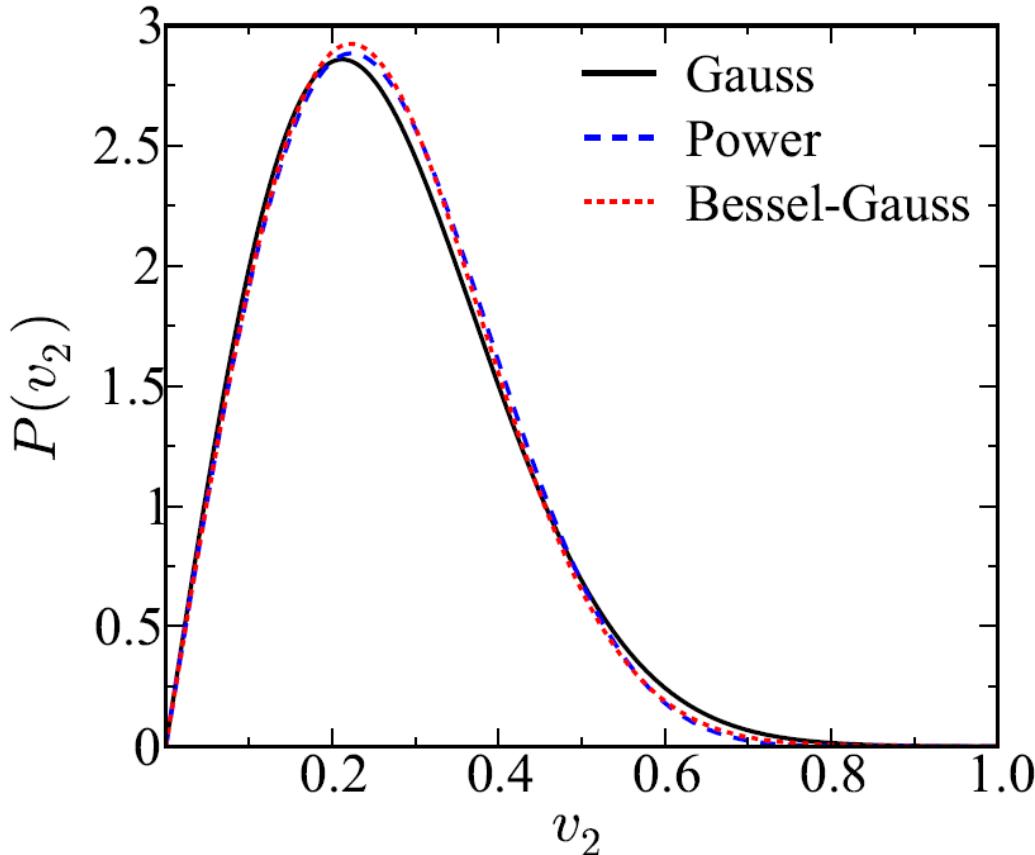
$$\begin{aligned}v_2\{2\} &\approx 0.295 \\v_2\{4\} &\approx 0.186 \\v_2\{6\} &\approx 0.169 \\v_2\{8\} &\approx 0.164\end{aligned}$$

# Glauber calculation for ellipticities



In A+A collisions  $P(\epsilon_2)$  is well described by Bessel-Gaussian distribution

$$P(\epsilon_2) = \frac{2\epsilon_2}{\sigma^2} I_0\left(\frac{2\epsilon_2 b}{\sigma^2}\right) e^{-(\epsilon_2^2 + b^2)/\sigma^2}$$



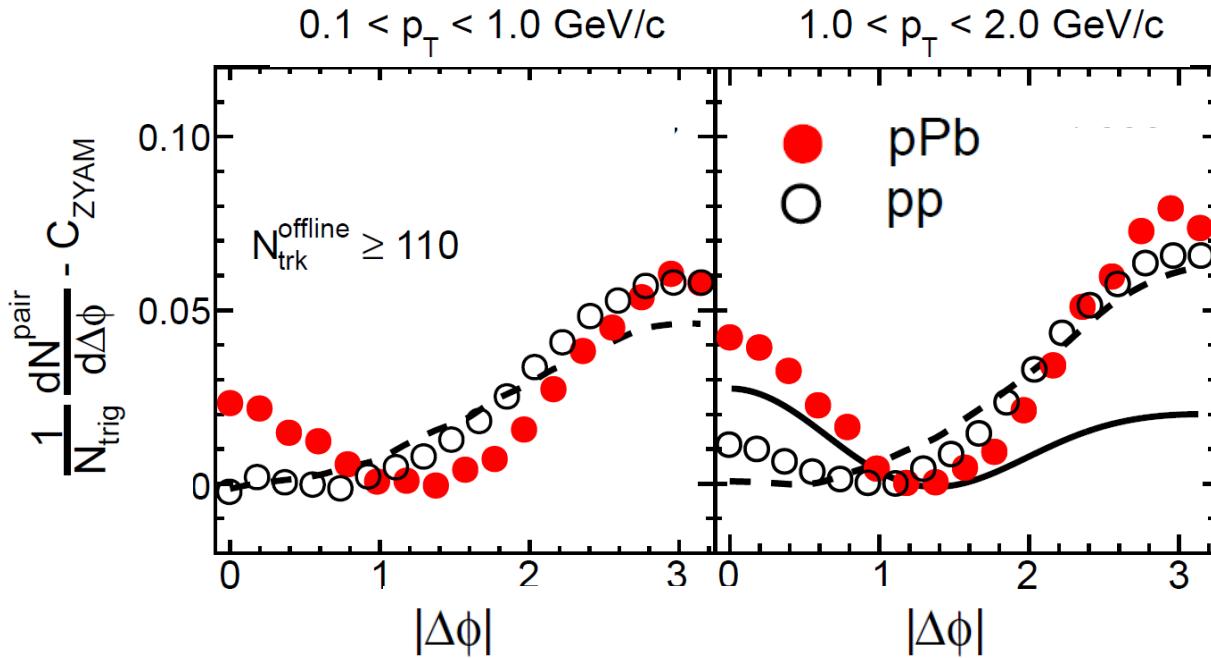
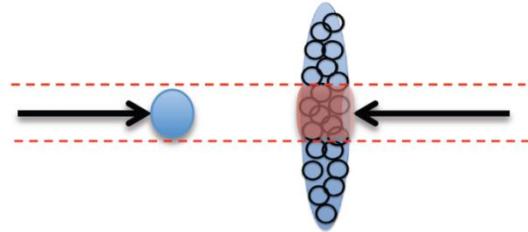
$$\begin{aligned}v_2\{2\} &= \sqrt{\sigma^2 + b^2} \\v_2\{4\} &= b \\v_2\{6\} &= b \\v_2\{8\} &= b\end{aligned}$$

In this plot:

$$\begin{aligned}v_2\{2\} &= 0.295 \\b &= 0.17\end{aligned}$$

## What about p+A

Proton-lead collisions at  $\sqrt{s} = 5020$  GeV



Color Glass Condensate ?  
Hydrodynamics ?

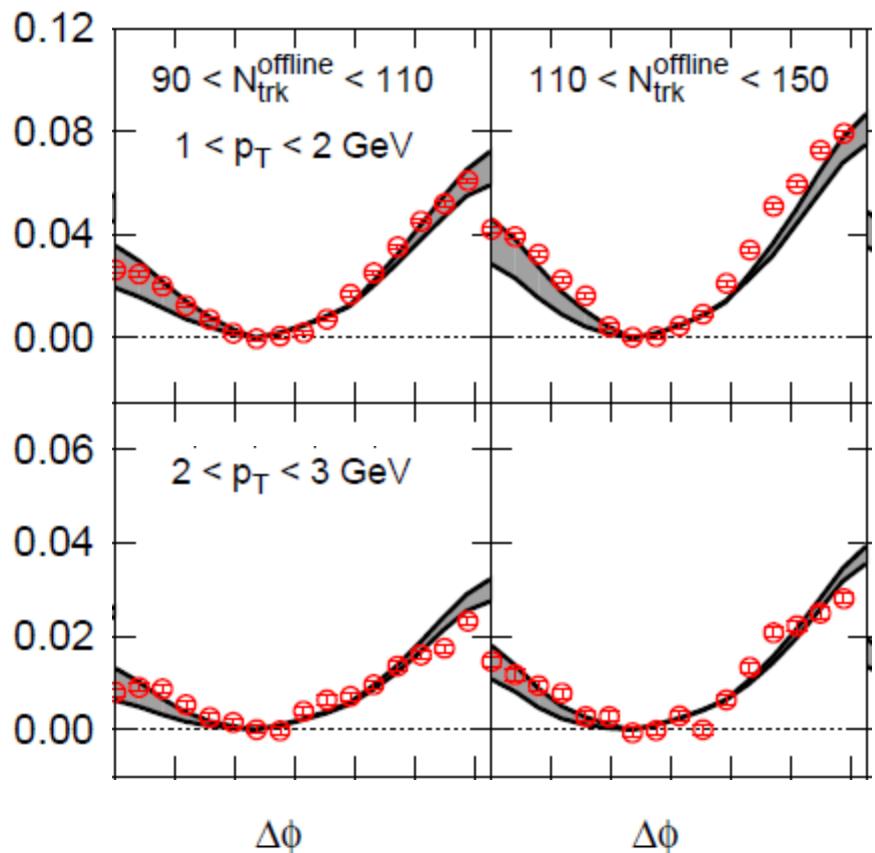
CMS Coll., Phys. Lett. B718 (2013) 795

ALICE Coll., Phys. Lett. B719 (2013) 29

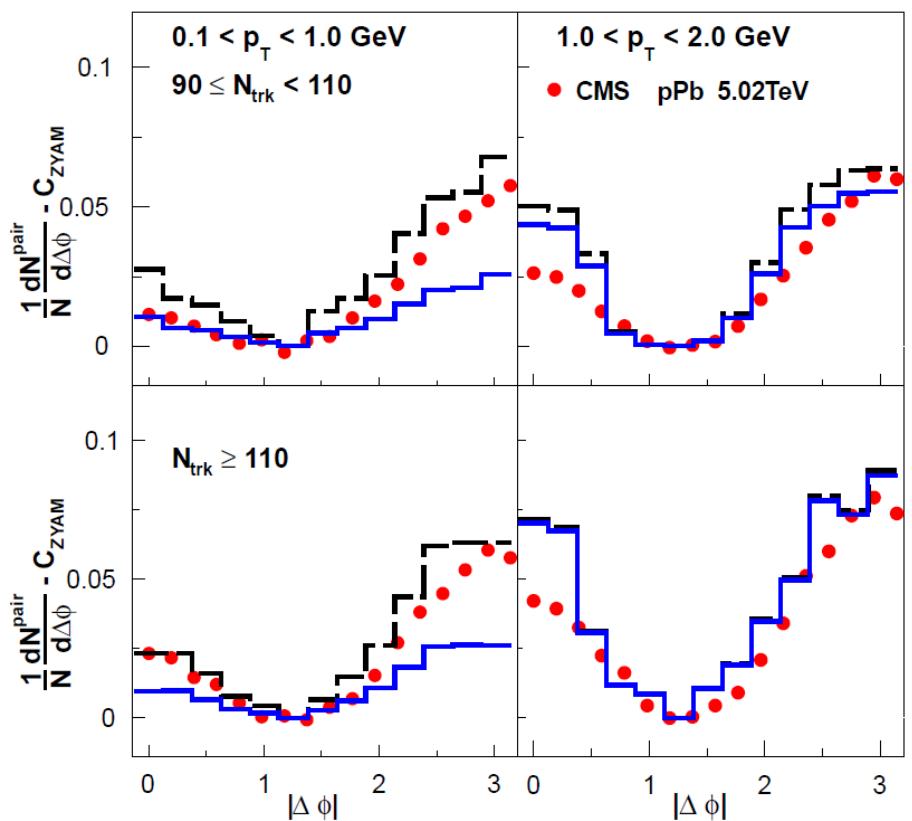
ATLAS Coll., Phys. Rev. Lett. 110, 182302 (2013)

PHENIX Coll., arXiv:1303.1794 [nucl-ex], d+Au at 200 GeV

# CGC (a sample)



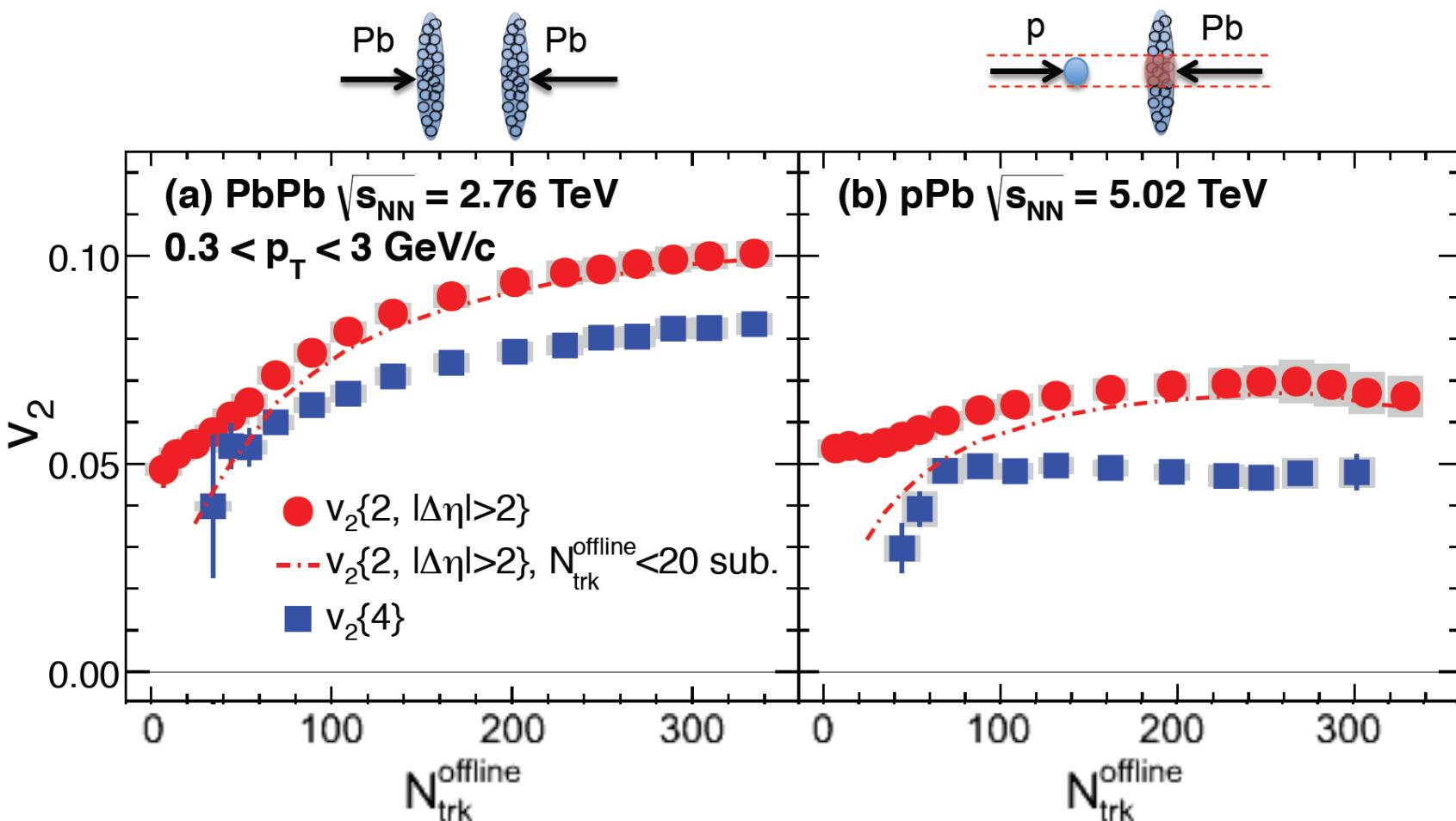
# Hydrodynamics



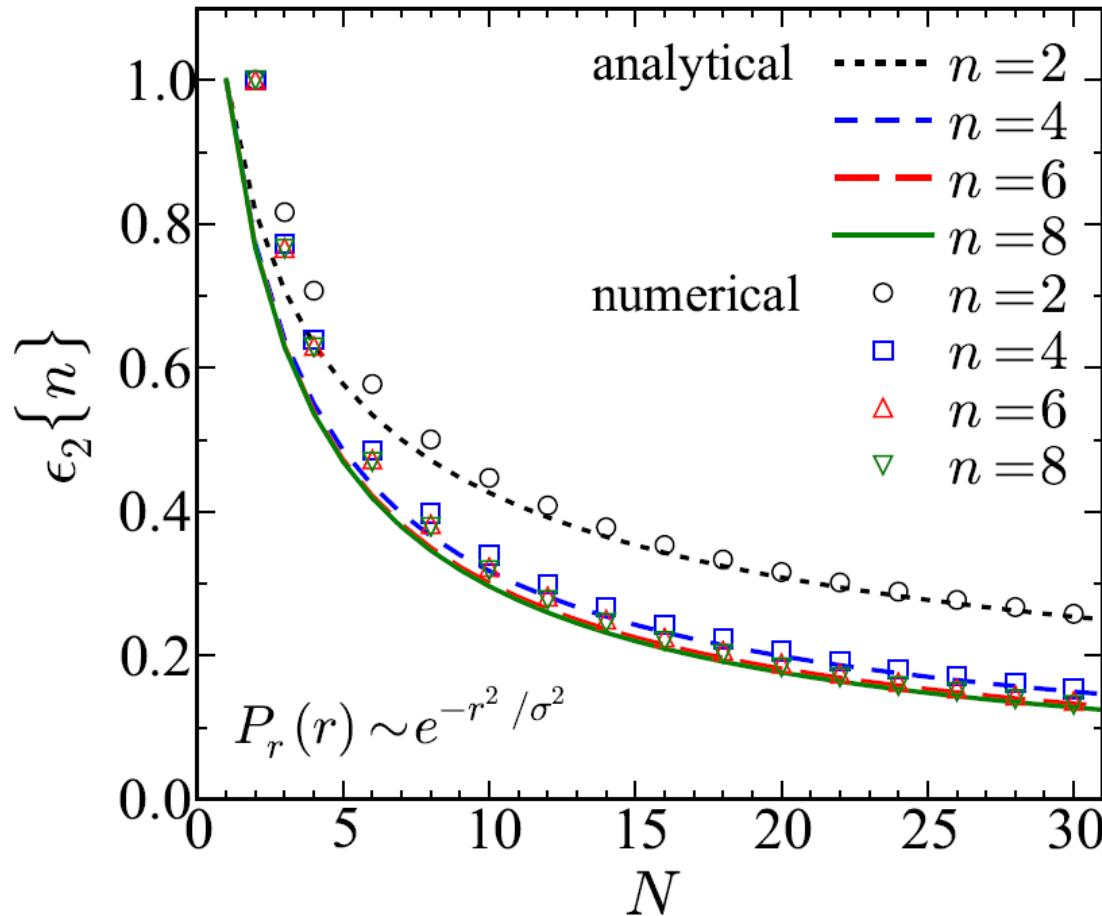
K. Dusling, R. Venugopalan, PRL 108 (2012) 262001,  
PRD 87 (2013) 094034

P. Bozek, PRC 85 (2012) 014911  
P. Bozek, W. Broniowski, PLB 718 (2013) 1557

# $v_2\{2\}$ and $v_2\{4\}$ in p+Pb



# Let's start with a random model



$$\langle \epsilon_2^{2n} \rangle = \frac{n! \lim_{z \rightarrow 0} \frac{d^n}{dz^n} \langle I_0(2\sqrt{z}r^2) \rangle^N}{\lim_{z \rightarrow 0} \frac{d^{2n}}{dz^{2n}} \langle e^{-r^2 z} \rangle^N}$$

AB, V. Skokov, arXiv:1312.7349

AB, P. Bozek, L. McLerran, arXiv:1311.7325

In p+A collisions  $P(\epsilon_2)$  is best described by power law distribution

$$P(\epsilon_2) = 2\alpha\epsilon_2(1 - \epsilon_2^2)^{\alpha-1}$$

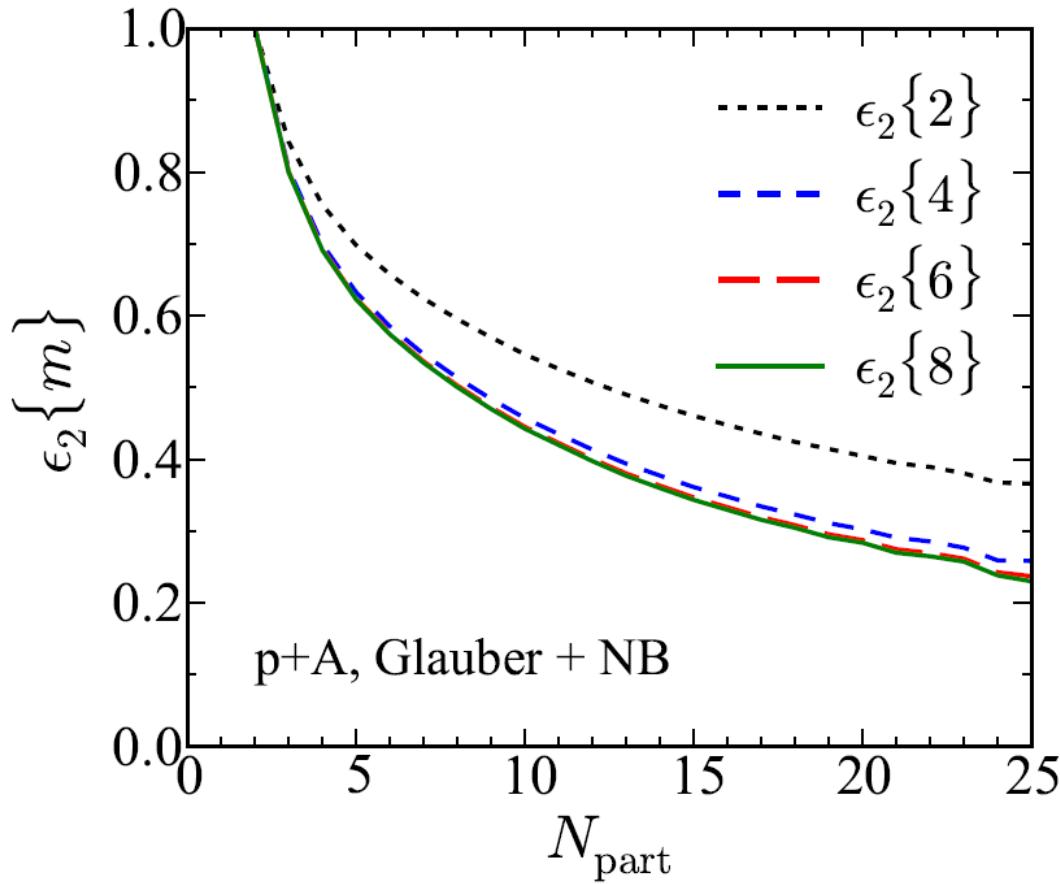
for Gaussian distribution  $P_r(r)$ :

$$\alpha = (N - 1)/2$$

$$\epsilon_2\{4\} = \epsilon_2\{2\}^{3/2} \left( \frac{2}{1 + \epsilon_2\{2\}^2} \right)^{1/4}$$

$$\frac{\epsilon_2\{4\}}{\epsilon_2\{2\}} = \left( \frac{2}{2 + \alpha} \right)^{1/4}$$

# Glauber in p+A with negative binomial distribution



$$\nu_2\{2\} = 0.082 \pm 0.002$$

Hydro calculations:  $\nu_2\{4\} = 0.055 \pm 0.004$

$$\nu_2\{6\} = 0.052 \pm 0.005$$

## Conclusions

- higher order flow cumulants are very sensitive to  $P(\epsilon_2)$
- in both A+A and p+A collisions we observe (Glauber)

$$\epsilon_2\{2\} > \epsilon_2\{4\} \approx \epsilon_2\{6\} \approx \epsilon_2\{8\}$$

- hydro evolution can translate it into

$$\nu_2\{2\} > \nu_2\{4\} \approx \nu_2\{6\} \approx \nu_2\{8\}$$

- any physics which translates initial geometry into final anisotropy should have this property
- CGC expectations are not yet clear (sign)